Variable density groundwater flow: are equivalent freshwater heads necessary or misleading?

by

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Variable Density Flow

A flow field containing fluids of different densities.

Velocity Potential [Muskat, 1937]

vs

Force Potential [Hubbert, 1940]

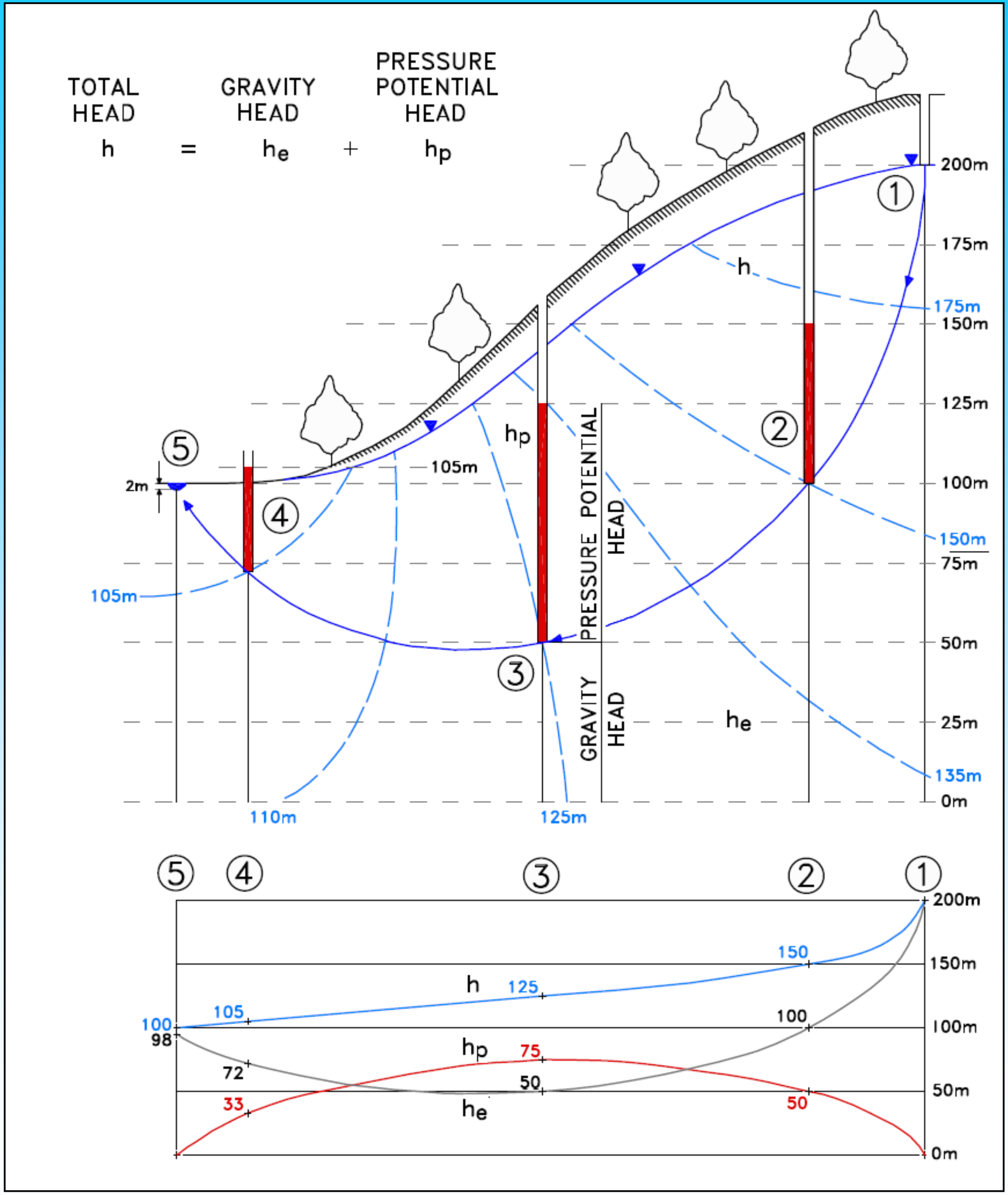
Velocity potential

1. Energy related to unit volume
2. Underground fluids therefore are assumed to be incompressible
3. Equivalent freshwater heads are assumed to stand for the actual energy conditions in a flow field

Force potential

1. Energy related to unit mass

Gravitationally-Driven Groundwater Flow



Unused energy stored by compressing the unit mass

Compression measured by water column in piezometers

after Weyer, 1978

Buoyancy

Hydrostatic vs Hydrodynamic

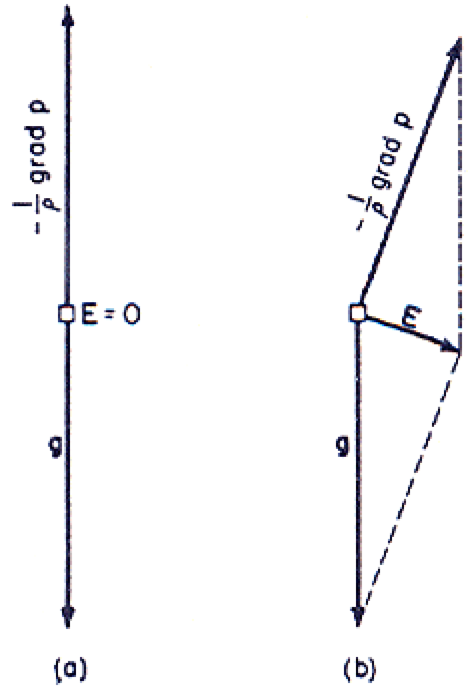
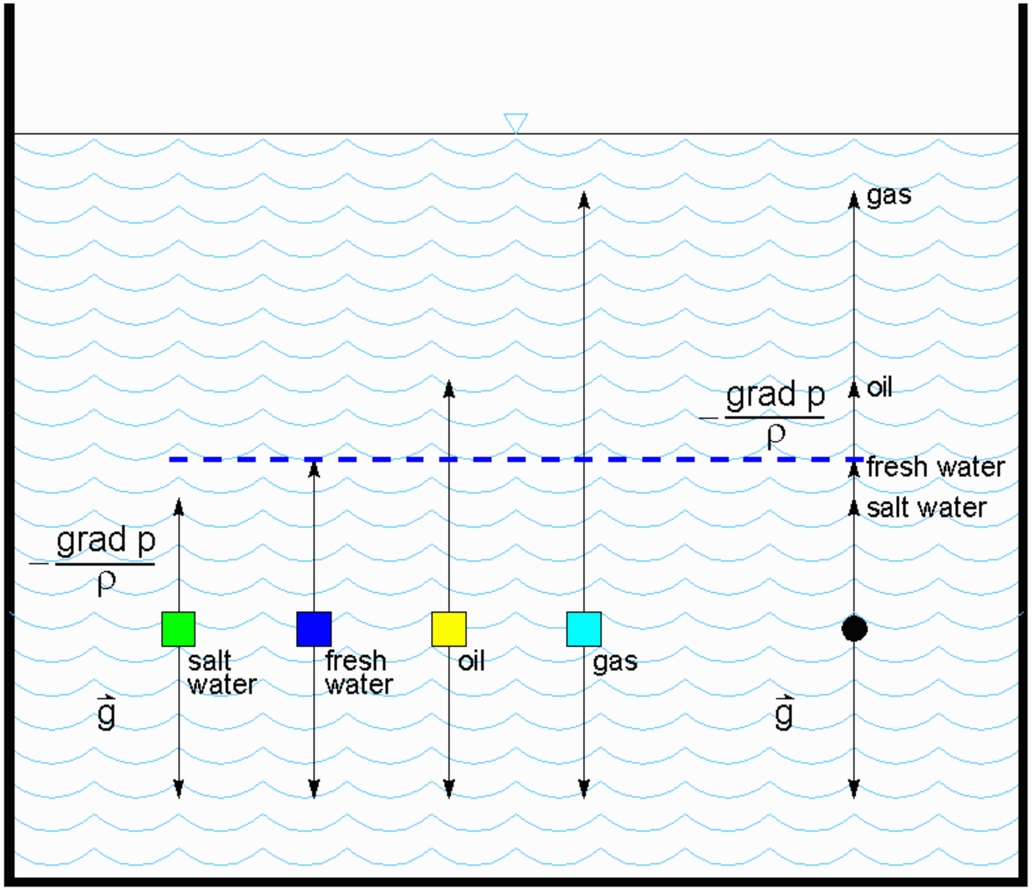


FIG. 4.—Physical interpretation of force-intensity vector **E**. (*a*) Hydrostatic case **E** = 0; (*b*) hydrodynamic case.

Hubbert, 1953. Entrapment of petroleum under hydrodynamic conditions



A = downward flow for dense salt water.... (grad p)/ρ < $\vec{g}$
B = no flow of fresh water......................... (grad p)/ρ = $\vec{g}$
C = upward flow for oil.............................. (grad p)/ρ > $\vec{g}$
D = upward flow for gas............................ (grad p)/ρ > $\vec{g}$

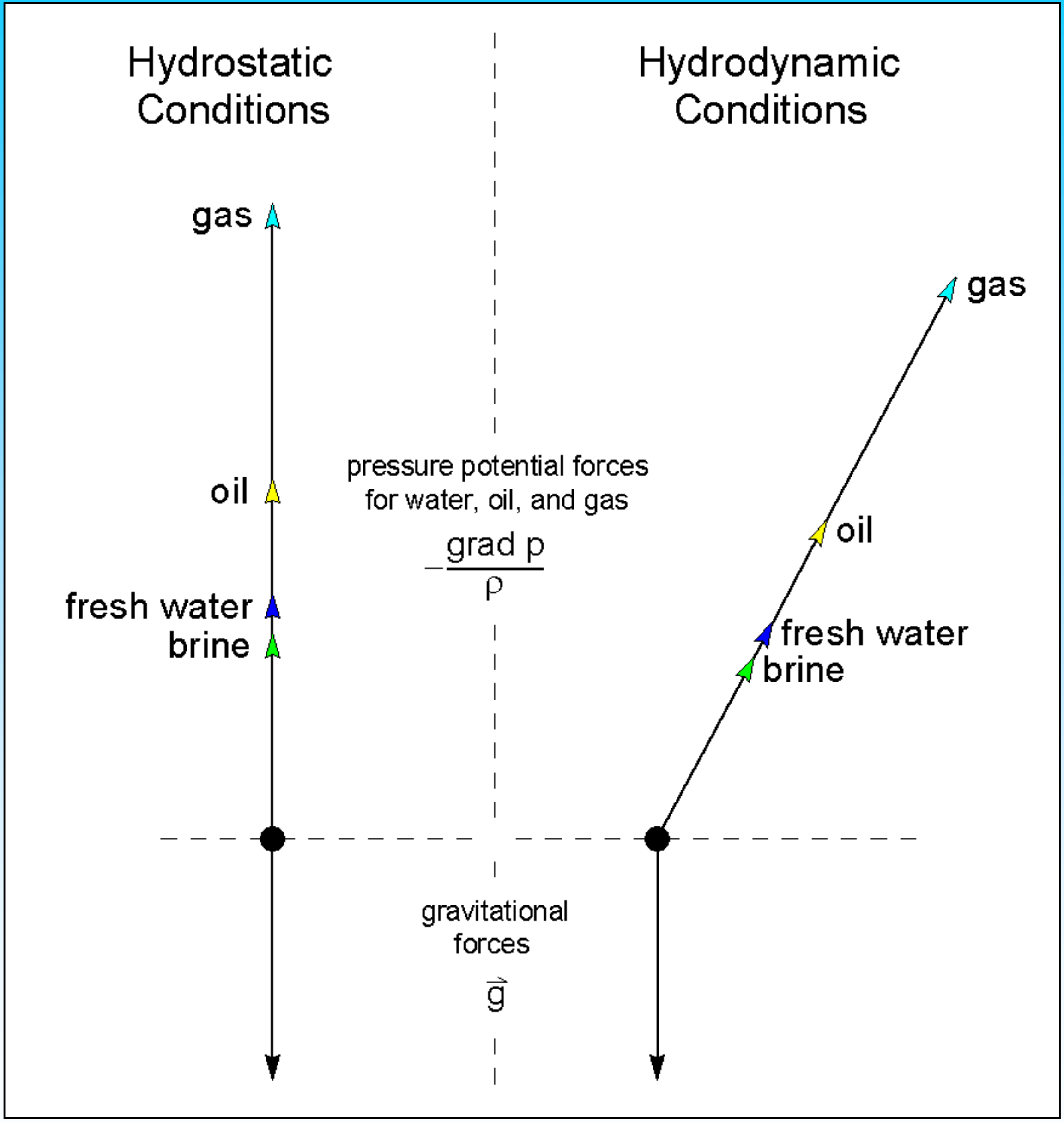
after Weyer, 2010

Schematic of hydrostatic forces on water, salt water, oil, and gas.

$\vec{g}$ = gravitational force

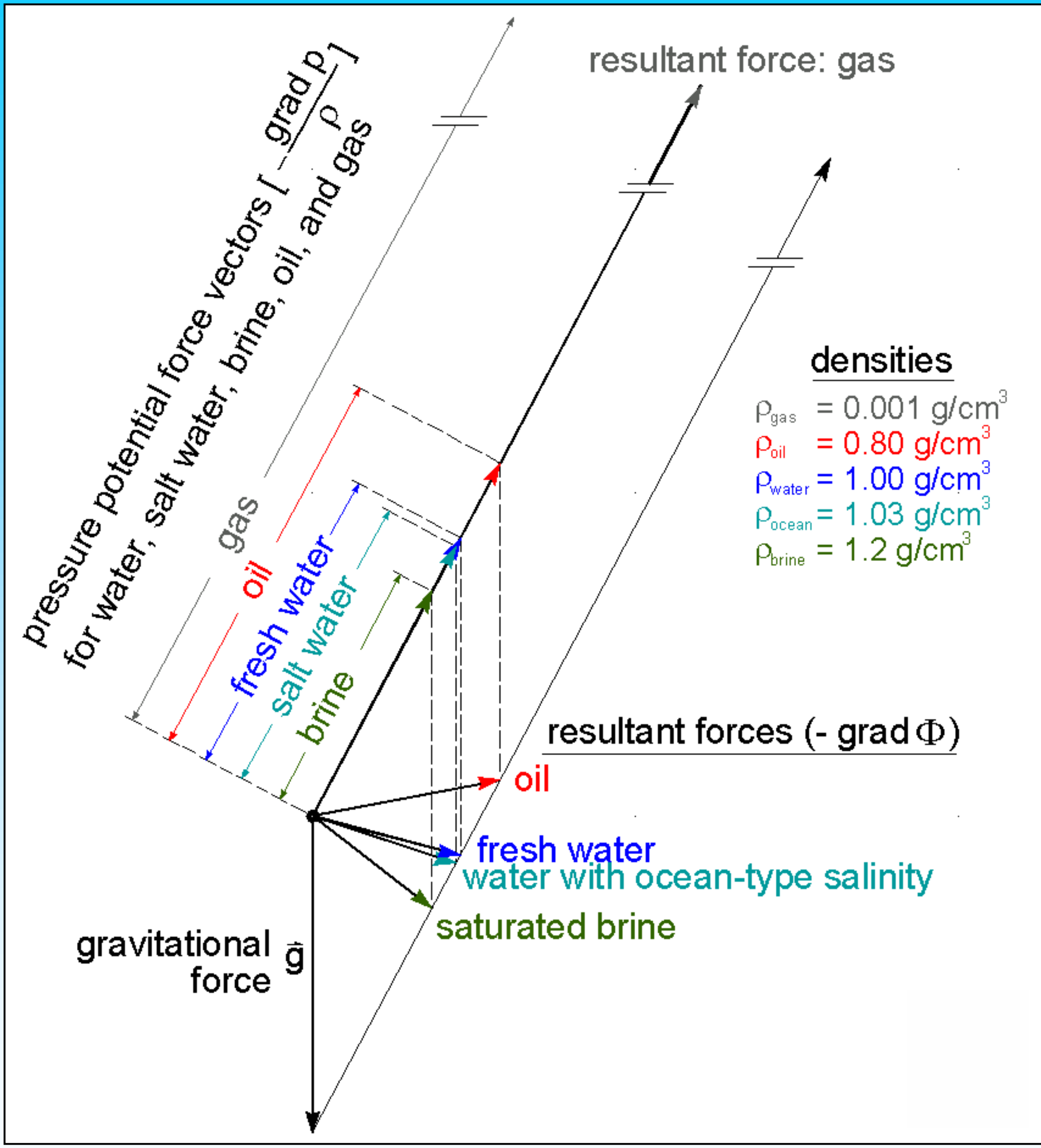
$-$(grad p)/ρ = pressure potential force (gradient of the pressure divided by the density)

buoyancy = $\vec{g}$ $-$ (grad p)/ρ



modified after Hubbert, 1953

Schematic of hydrostatic vs hydrodynamic forces on water, oil, brine, and gas.

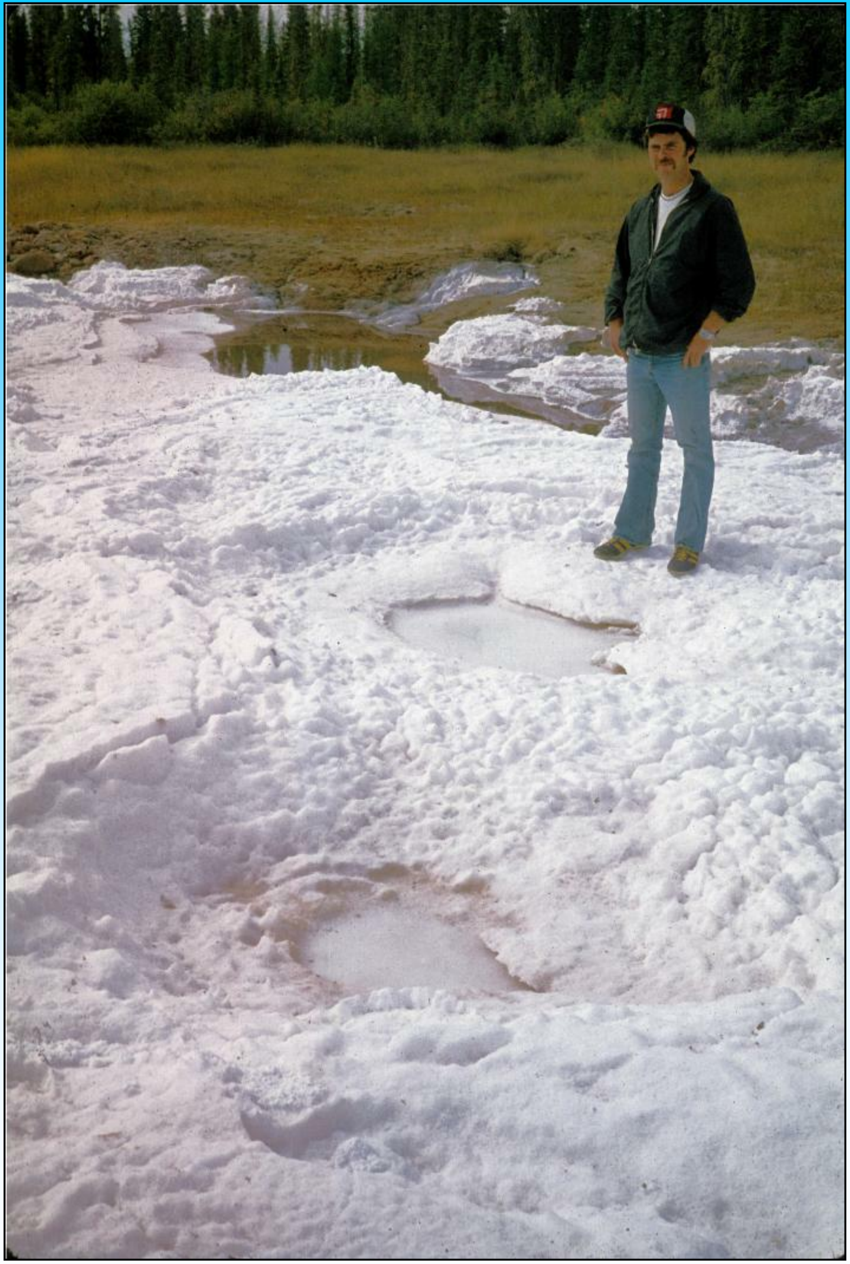


modified after Hubbert, 1953

Schematic determination of differing flow directions for fresh water, salt water, brine, oil, and gas within the same fresh water force field based on density differences.

Field examples for variable density flow

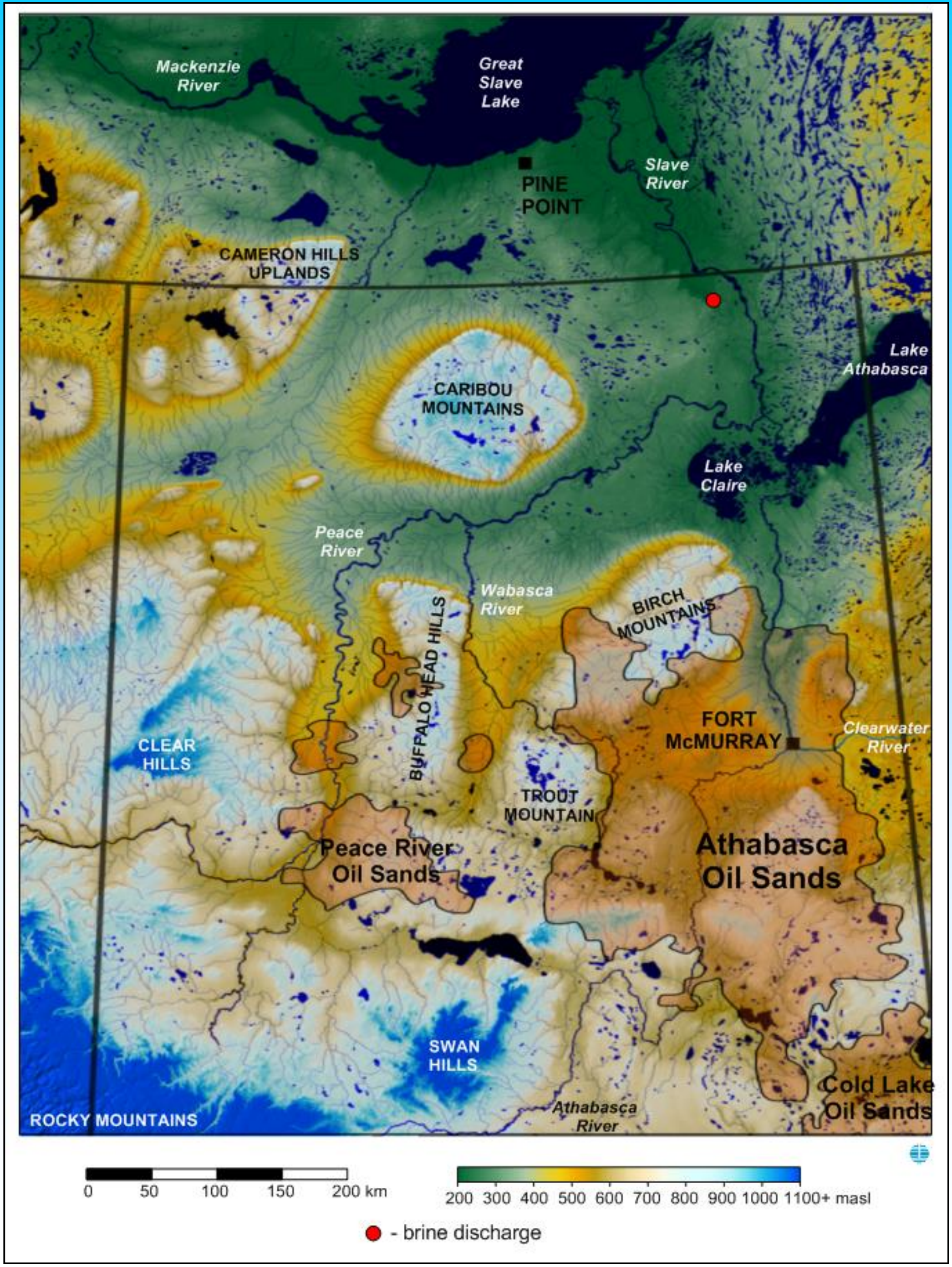
Field example Salt River Basin, NWT, Canada:
Upward discharge to the surface of <u>saturated brine</u>.



Upward migration of saturated brine: Salt River Basin, NWT, Canada

- Saturated [~350 g/l; density ~1.3 g/cm^3] brine discharging upwards beside a creek
- Salt deposit is caused by precipitation of salt from brine not by evaporation of brine.

picture: K.U.Weyer, 1977



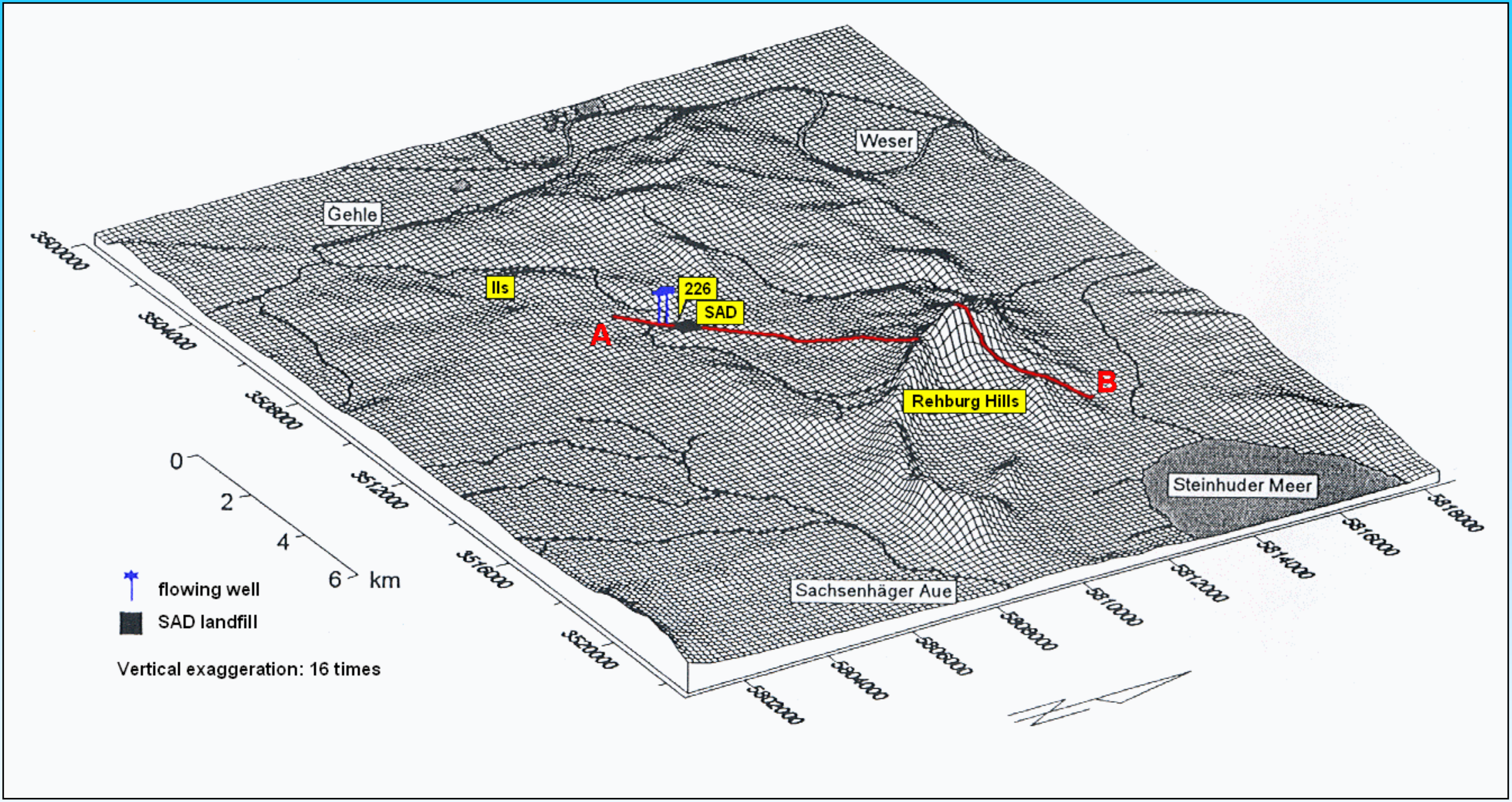
Upward discharge of saturated brine to the surface in the head waters of the Salt River

Extent of oil sands taken from Einstein, 2006

According to many scientists and practitioners worldwide, downward-directed buoyancy forces should prevent this from happening.

‘Density changes’ along flow lines within a real-world groundwater flow system

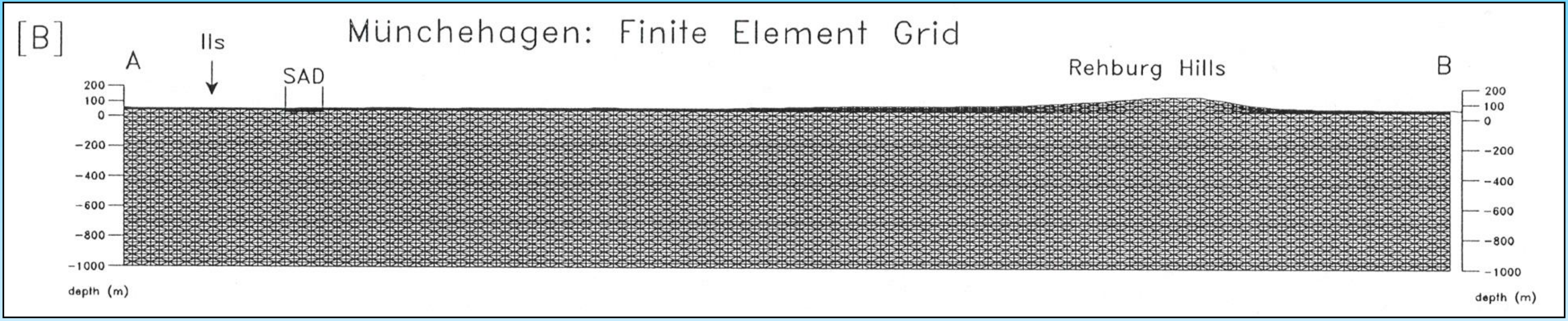
Field example and mathematical model: Upward discharge from depth of 1000 m of **ocean-type saline water** at Münchehagen, Germany



after Weyer, 1996.

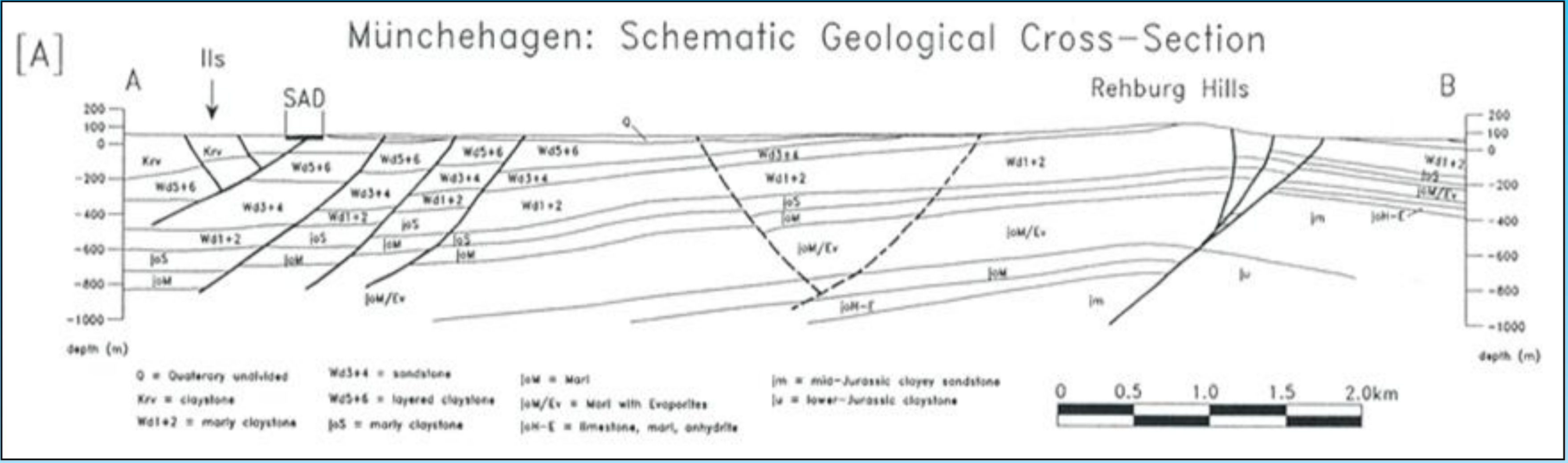
Industrial landfill Münchehagen: DEM, location of cross-section A-B, landfill (SAD) and borehole 226.

2D-vertical model of groundwater flow directions at the Münchehagen landfill area



Weyer and van Everdingen, 1995, Fig.3

Calculated groundwater flow directions based on groundwater table (following topography), geological structure, and estimated permeability contrasts. Program: FLONET



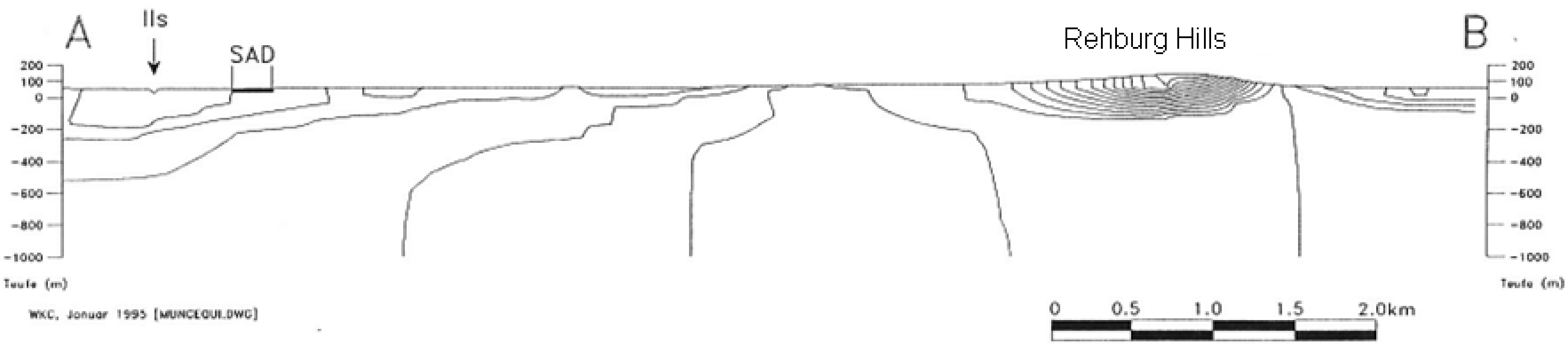
Weyer and van Everdingen, 1995, Fig.3 and Tab.1

Table 1: Geology and assigned contrast permeabilities near Münchehagen

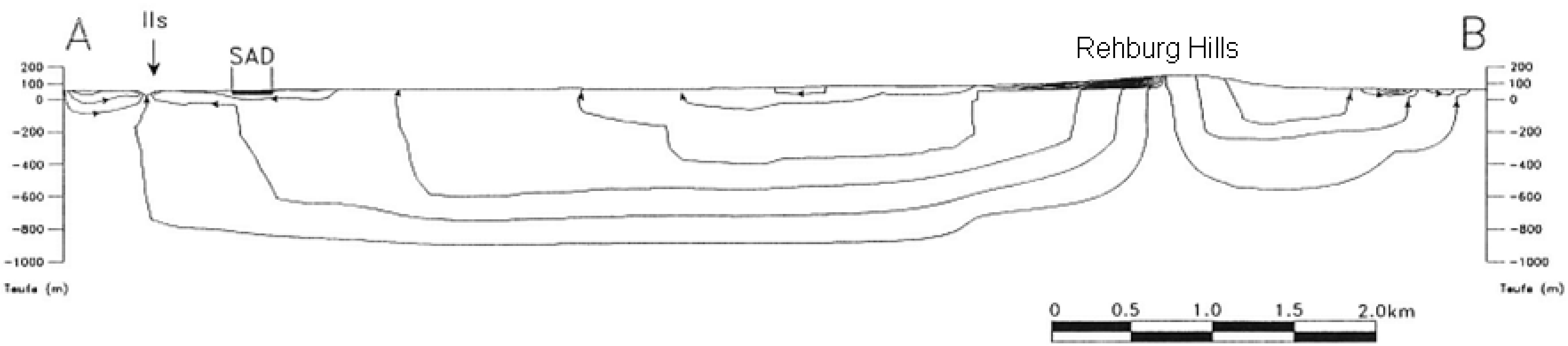
Formation Name	Abbr.	Geology		Permeability Eastern half Loccum	Permeability Western half Rehburg
Quarternary	Q	sand, cobbles		--	--
Cretaceous	Krv	claystone	5-15m	200	--
	"		15-50m	100	--
	"		deeper Krv layers	50	--
Wealdon 1, 2	WD 1+2	marly claystone		5	1
Wealdon 3, 4	WD 3+4	sandstone		10	100
Wealdon 5, 6	WD 5+6	layered claystone		1	1
Serpulit	joS	marly claystone		10	10
Mündener Mergel	joM	marl		100	100
Mündener Mergel	joM/Ev	marl with evaporites		100	100
Heersumer	joH-E	limestone, marl, anhydrite		100	100
mid-Jurassic	jm	clayey sandstone		10	10
low-Jurassic	ju	claystone		10	10

Geologic cross-section taken from official geologic maps 1:25,000 of the state of Lower Saxony, contrast permeabilities assigned by us.

Münchehagen: Equipotential Lines

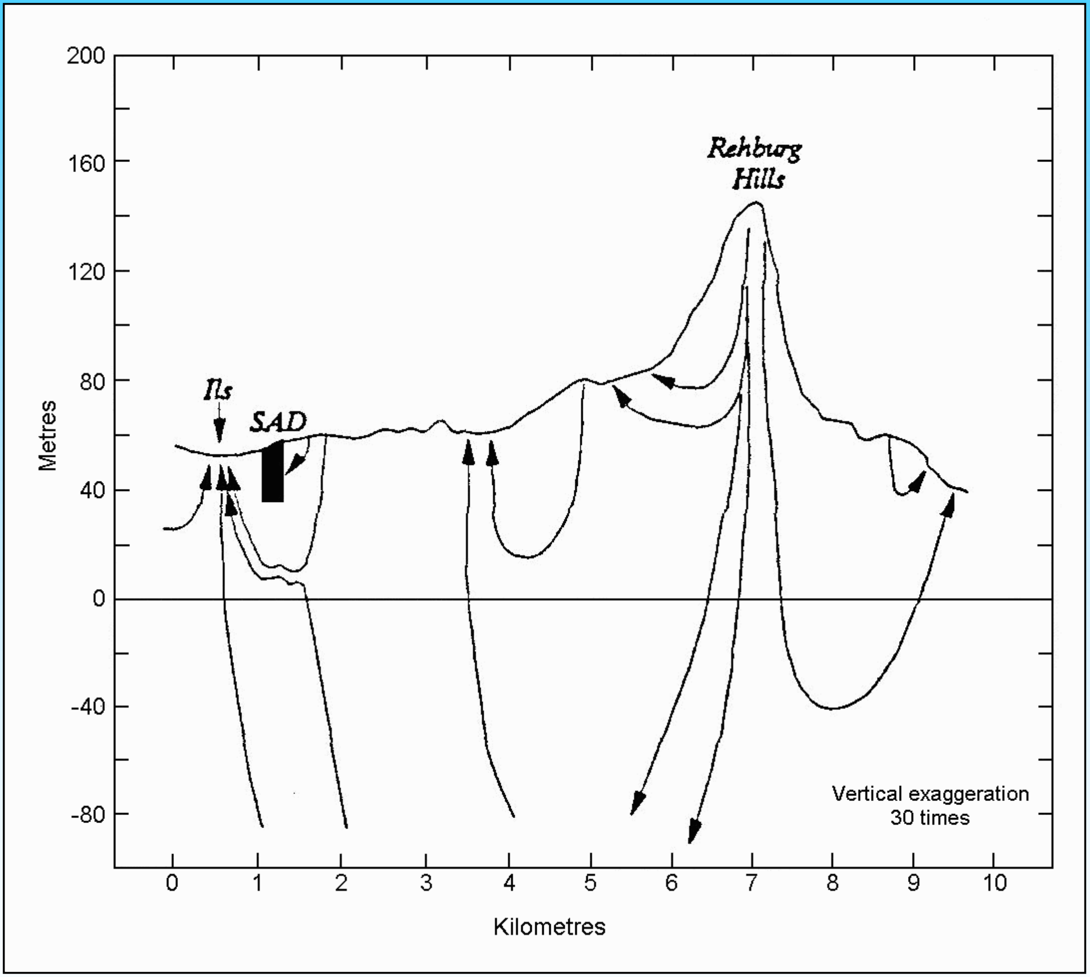


Münchehagen: Flow Lines



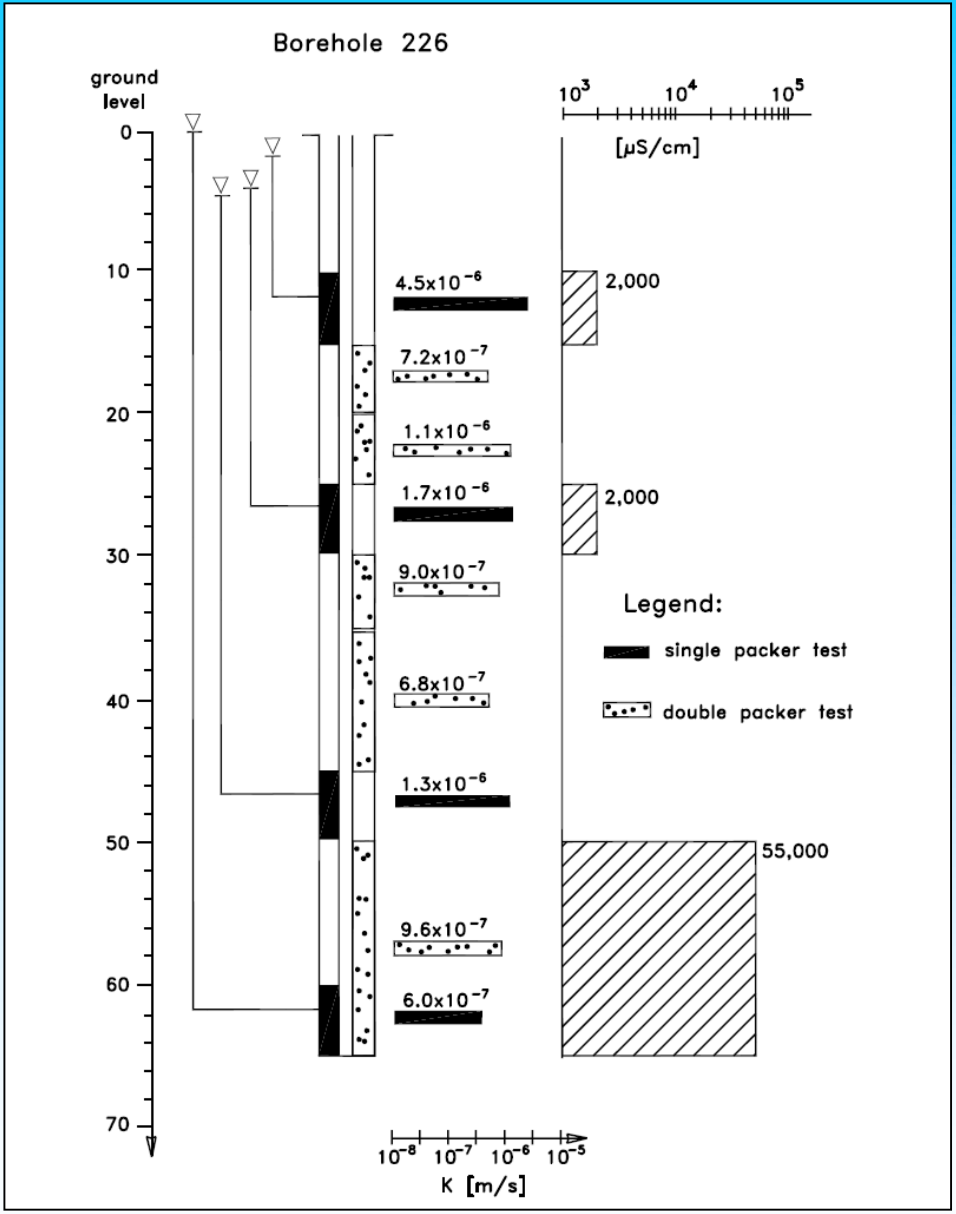
after Weyer, 1996, Fig.74

Cross-section A-B showing flow lines with enhanced exaggeration (30:1) calculated by 2D-vertical mathematical model



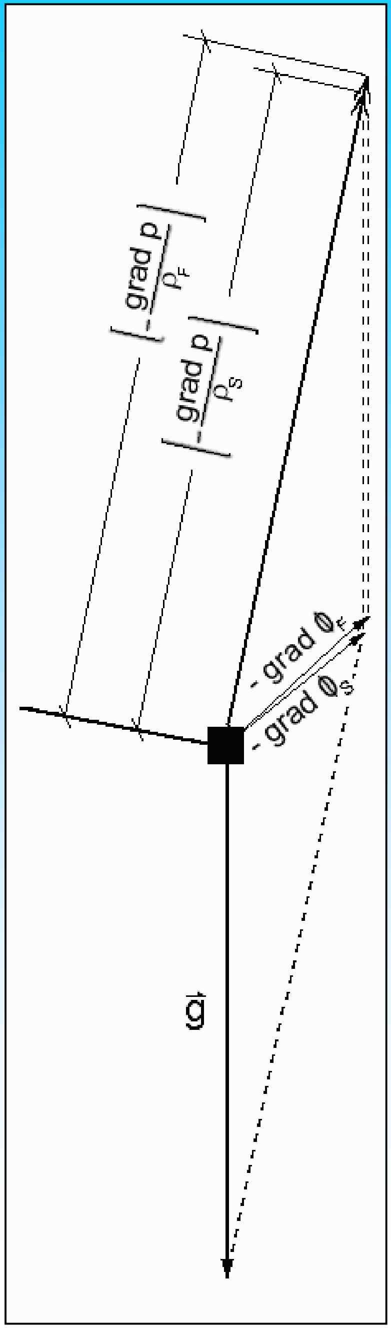
after Weyer, 1996

- SAD = landfill Münchehagen
- Laterally-compressed flow lines as returned by model calculation; vertical exaggeration 30:1
- Upward flow line of saline water occurs at 50 m depth below ground



Gronemeier et al., 1990, Fig.7

Occurrence of ocean-type saline water in borehole 226 at a depth of about 50 m below ground.



In this case fresh water modeling proved to be suitable to determine the flow lines of saline seawater

Density of salt water ρ = 1.03 g/cm^3

Weyer, 1996

Traditional ways of looking at variable density flow

The big difference between assumptions and physical reality – or – where the handling of variable density flow usually goes astray

Referring to Bear (1972, p. 654), Bachu et al. (1993, p. 7) and Bachu and Underschultz (1993, p.1754) both considered two types of driving forces for variable density groundwater flow in northeast Alberta, "*a **potential force resulting from piezometric head differences**, and a **buoyancy force** resulting from density differences (Hubbert, 1940; Bear, 1972)*".

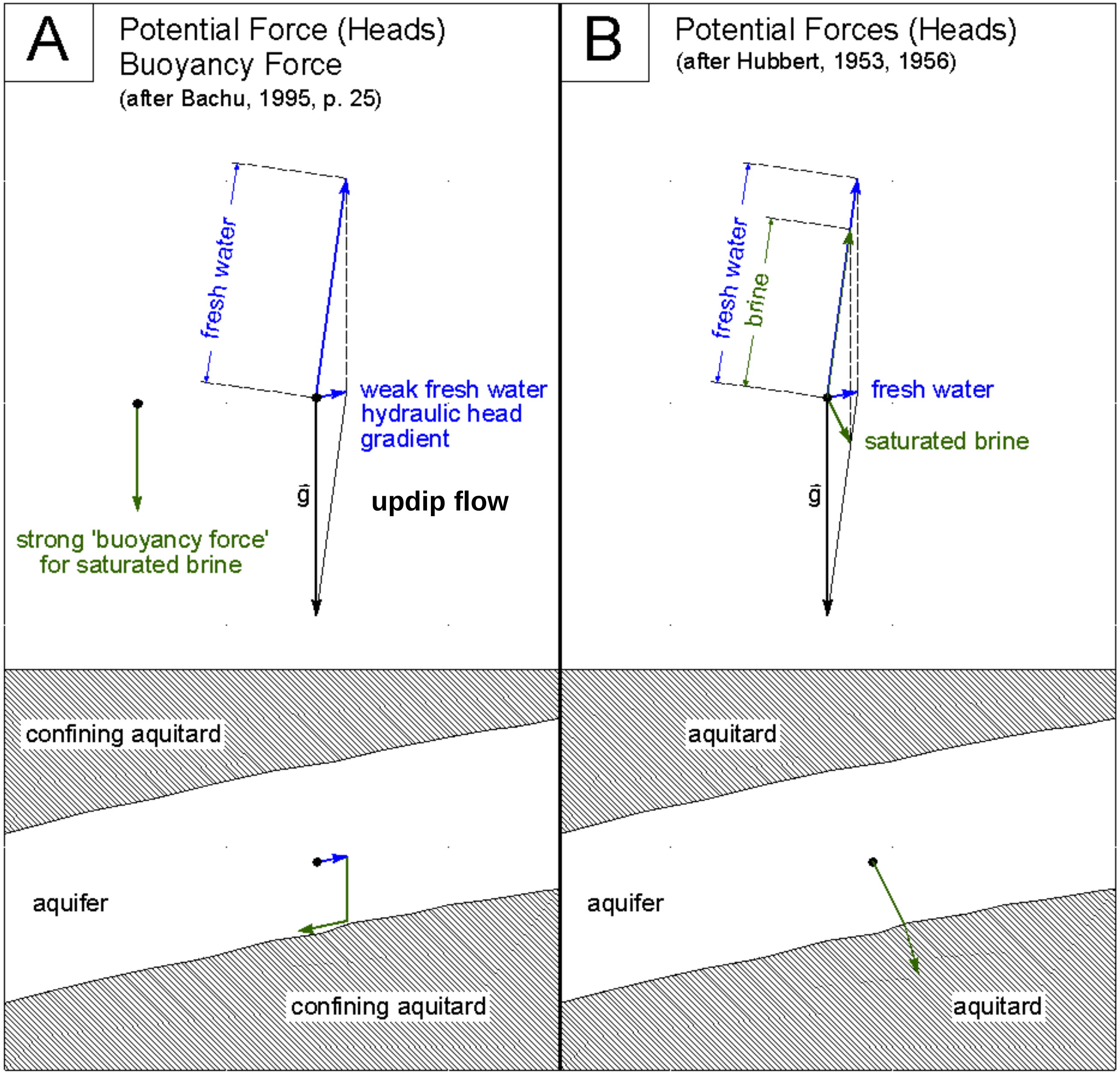
<u>False quote: Hubbert (1940) does not refer to buoyancy forces or density differences</u>

Bear, J., 1972. Dynamics of Fluids in Porous Media. American Elsevier Publishing Company, Inc., New York, NY, USA, 764p., ISBN: 978-0444001146

Bachu, S., J.R. Underschultz, B. Hitchon, and D. Cotterill, 1993. Regional-Scale Subsurface Hydrogeology in Northeast Alberta. Alberta Research Council, Bulletin No. 61, 44 p.

Bachu S., and J.R. Underschultz, 1993. Hydrogeology of formation waters, northeastern Alberta basin. AAPG Bulletin, vol. 77, issue 10, p. 1745-1768.

Bachu, S., 1995. Flow of variable-density formation water in deep sloping aquifers: review of methods of representation with case studies. Journal of Hydrology, vol. 164, p. 19-38.



Schematic comparison of hydraulic forces by Bachu (1995) and Hubbert (1953, 1956).

Bachu (1995) clearly lays out the conditions for buoyancy forces that had already been described in Bachu et al. (1993) and Bachu and Underschultz (1993).

Weyer and Ellis, 2015

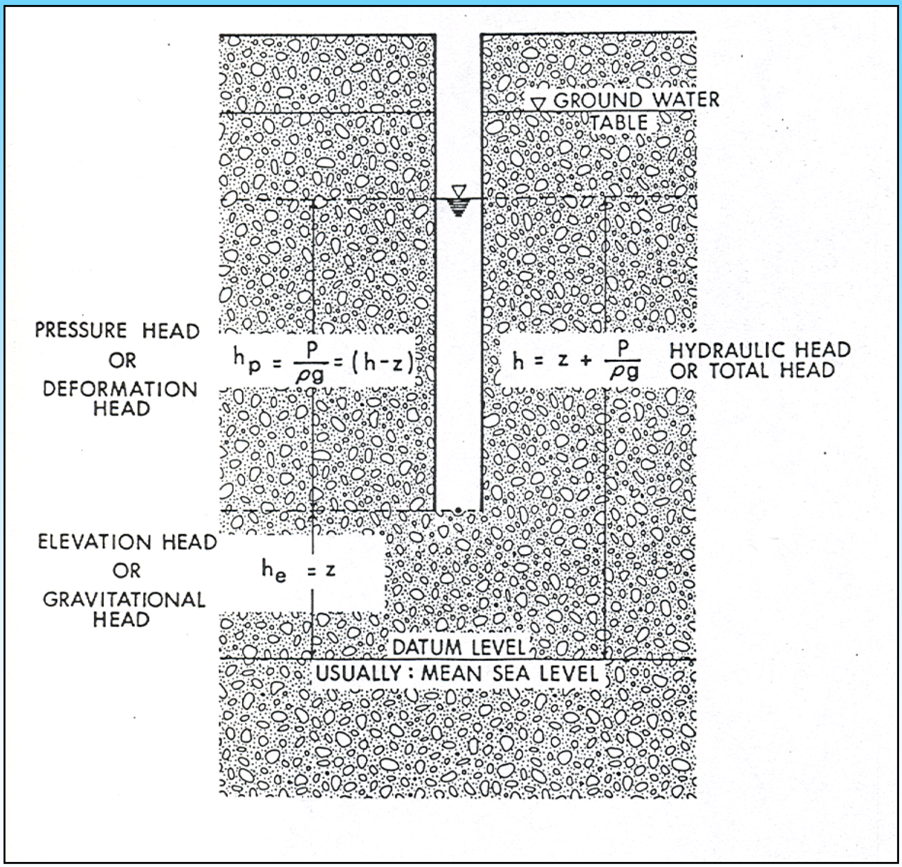
Equivalent freshwater head

A widely used attempt to determine the state of energy as a fresh water head within groundwater bodies of variable density

Heads

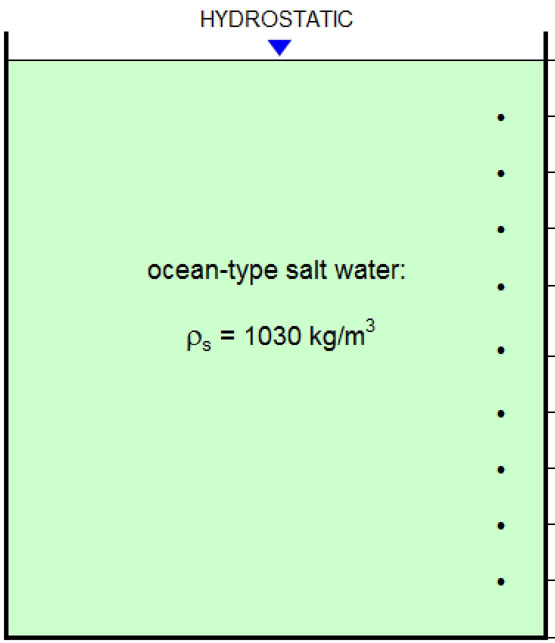
$$\Phi = h * g$$

potential energy = head • earth acceleration



after Weyer, 1978

- hydraulic head = elevation of water level in a piezometer (above datum level)
- gravitational head = screen elevation in gravitational field
- pressure head = height of water column in a piezometer



Elevation z [m]	Pressure P [Pa]	Equiv. Fresh Water Head $H_f = z + P/\rho_o g$ [m]	Gravitational Potential $\Phi_g = Z^*g$ [m^2/s^2]	Pressure Potential $\Phi_p = P/\rho_s$ [m^2/s^2]	Total Force Potential (energy/mass) $\Phi = \Phi g + \Phi p$ [m^2/s^2]	Elevation Head $H_e = \Phi_g/g$ [m]	Pressure Head $H_p = \Phi_p/g$ [m]	Total Head $H_t = \Phi/g$ [m]
10	0	10	98.1	0	98.1	10	0	10
9	10104.3	10.03	88.29	9.81	98.1	9	1	10
8	20208.6	10.06	78.48	19.62	98.1	8	2	10
7	30312.9	10.09	68.67	29.43	98.1	7	3	10
6	40417.2	10.12	58.86	39.24	98.1	6	4	10
5	50521.5	10.15	49.05	49.05	98.1	5	5	10
4	60625.8	10.18	39.24	58.86	98.1	4	6	10
3	70730.1	10.21	29.43	68.67	98.1	3	7	10
2	80834.4	10.24	19.62	78.48	98.1	2	8	10
1	90938.7	10.27	9.81	88.29	98.1	1	9	10
0	101043	10.3	0	98.1	98.1	0	10	10

after Weyer and Ellis, 2015

Determination of equivalent freshwater heads in a tank filled with ocean type salt water. According to the equivalent freshwater head calculation the head at the bottom of the tank (10.3 m) is much higher than at the water surface (10 m) and upward flow must occur, which it does not. On the right of the table the correct head calculations are recorded according to force potential procedures confirming hydrostatic conditions with a head of 10 m throughout.

What we addressed:

- Hydrostatic vs hydrodynamic flow conditions (buoyancy forces).
- Bachu and Underschultz's (1993) buoyancy forces do not exist.
- In groundwater discharge areas, saturated brine can flow upwards to the surface.
- At the industrial landfill site Münchehagen variable density flow transports ocean-type salt water to the small river Ils. The respective flowlines were successfully calculated in a numerical model of a groundwater flow system using a density ρ =1 g/cm^3.
- Equivalent fresh water heads do not represent actual energy conditions in a variable density flow field and are misleading.

Force Potential
[energy/mass]

- Mass is measured in kilograms.
- A kilogram is independent of pressure, density, and temperature.
- Heads measured in piezometers of **any** pressure, density, temperature, etc., are the correct representation of energy if flow calculations and computer modelling are done with force potentials.
- Calculations and computer models with velocity potential lead to incorrect results in variable density flow.

Synopsis

- When dealing with variable density flow under on-shore hydrodynamic conditions the application of physically-consistent force potentials and groundwater flow systems theory are the methods of choice.
- Computer programs making use of velocity potential (SUTRA and others) need to be surpassed by programs built on force potential. These programs do not yet exist.